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NON-INVASIVE MEASUREMENT. STRIKING ACCURACY.

TEST REPORT

On the Precision of Gamma-Ray and Percussion (MPD) Densitometers for Loose Solid Materials - A. Raykhman

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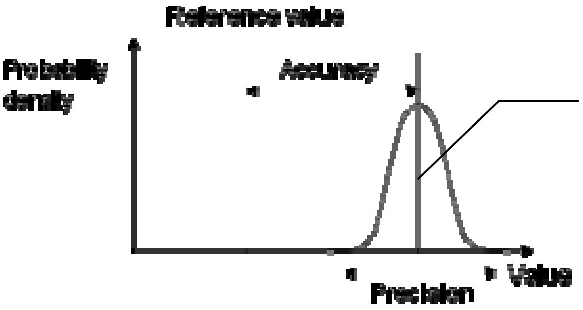
A substantial gap exists between conventional understanding of the accuracy of a measuring device and its metrological definition. The former describes validity of the measuring device mostly focusing in the ability to numerically evaluate a physical variable reference value at any given moment of time. Such definition combines two separate characteristics of the measuring device as they defined by the theory of measurement [1]:

- Accuracy is the degree of veracity, or more detailed, the degree of closeness of a measured or calculated quantity to its actual (true) value
- Precision is the degree of reproducibility or repeatability meaning the degree to which further measurements or calculations show the same or similar results

Validity = Accuracy & Precision

The above-made definitions are illustrated by a Figure 1 drawing.



Mean of a representative sample of readings produced by a measuring device (Mean value)

Figure 1.

Frequently, accuracy is evaluated by the absolute or relative bias:

Bias = |Mean value – Reference value) or

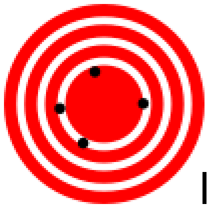
Bias = 100%·|Mean value – Reference value)|/Reference value

Bias is a non-random or directed effects caused by a factor or factors unrelated by the independent variable [1].

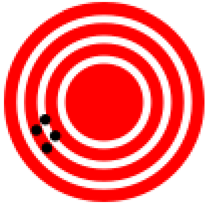
The bias can be eliminated by calibration; therefore the accuracy of the measuring device is definable by the accuracy of a standard measuring device used in the calibration procedure.

Consequently, precision is described by random error of measurement that could be represented by a standard deviation obtained on the representative measurement sample.

To better understand the difference between the accuracy and precision, a target analogy could be used shown in the Figure 2 below.



High **accuracy**, but low **precision**



High **precision**, but low **accuracy**

Figure 2.

Often, such terminological misuse can be observed in scientific or engineering articles including those cited in this paper. Further, each incident of imprecise use of measurement terminology will be marked by the author of this paper.

1. Precision of the Gamma-Ray Method

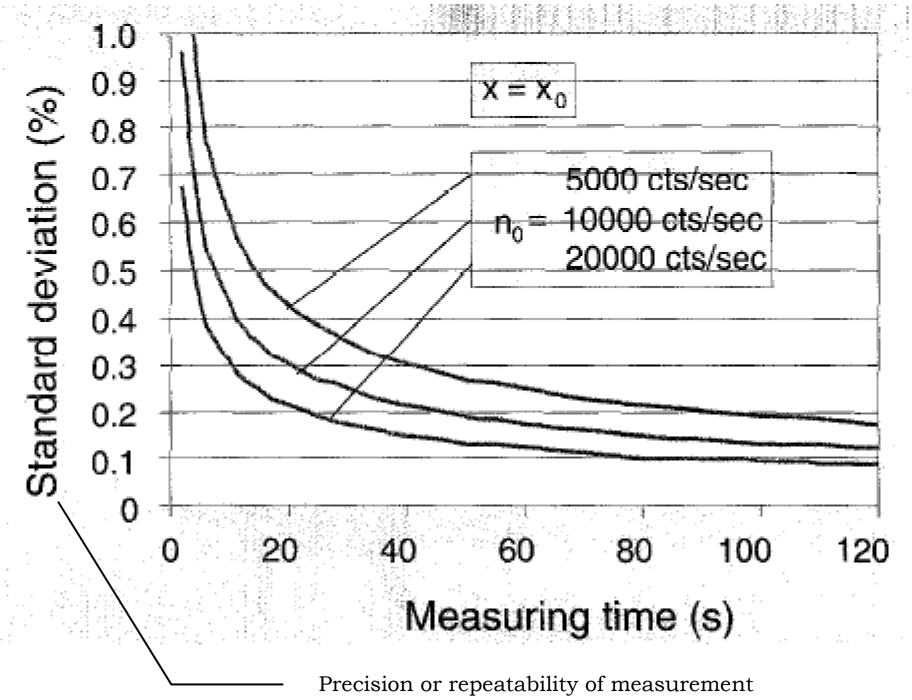
Supposition

An amount of radioactive energy (number of gamma rays) produced by a source matches the type of a material which density is supposed to be measured. Therefore, the accuracy loss due to an incorrect path length or material density range defining the attenuation coefficient of a gamma-ray device is not a subject of the present analysis [2].

According to Dr. Georg Schlieper [3] "…A sufficient amount of counts *(number N of gamma rays registered by a scintillating chamber of the device [A.R.]*) is required for a high accuracy of measurement… The accuracy* of results following a Gaussian probability distribution is defined as the standard deviation ($\sigma(N)$) in relation to the measured value":

$$\begin{gathered}\sigma(N)=\sqrt{N}\\ \frac{\sigma(N)}{N}=\frac{1}{\sqrt{N}}\\ N-number\ of\ counts\end{gathered}\tag{0.1}$$

The accuracy* increases by the increase of the accumulated number of counts, hence the accuracy* is directly proportional to the measuring time, $N = n\tau$, where τ denotes the measuring time and *n* denotes the counts rate. A Figure 3 graphs borrowed from [3] illustrate the relationship between the measured time and the accuracy* of gamma ray density measurement. Here, x_0 represents the optimal path length in accordance with the above-made supposition.



Precision or repeatability of measurement

Figure 3.

*Incorrect terminology: Precision must be used instead /A.R./
Dr. Shlieper concluded his article by giving the following values for accuracies of gamma ray densitometers working in favorable conditions:

"…the **accuracy*** of measurement can be less than 0.2% after a measuring time of 60 seconds. Practical standard deviations are **usually between 0.3 and 0.5 %**."

Other authors suggest a similar estimation. For example, using parameters and formulas presented in the article published by Glenn O. Brown [2], one can analyze the light powder density measurement accuracy (Evonik test) with the following results.

$$I = I_0 e^{-\mu(\rho)s} \tag{0.2}$$

Where I denotes the number of gamma rays registered by the scintillation chamber; I_0 denotes the number of gamma rays emitted by the source; μ denotes the density-dependent attenuation factor $\equiv$ linear attenuation coefficient; *s* denotes the path length.

Given *s=50 cm*, $\rho \in [20{,}150]\, g/L$, $\tau = 60s$, *n=1000*; using the standard deviation of attenuation graph [2] shown in the Figure 4, and extrapolation of the linear attenuation coefficient for carbon [4] presented in the Figure 5, one could receive the following precision value:

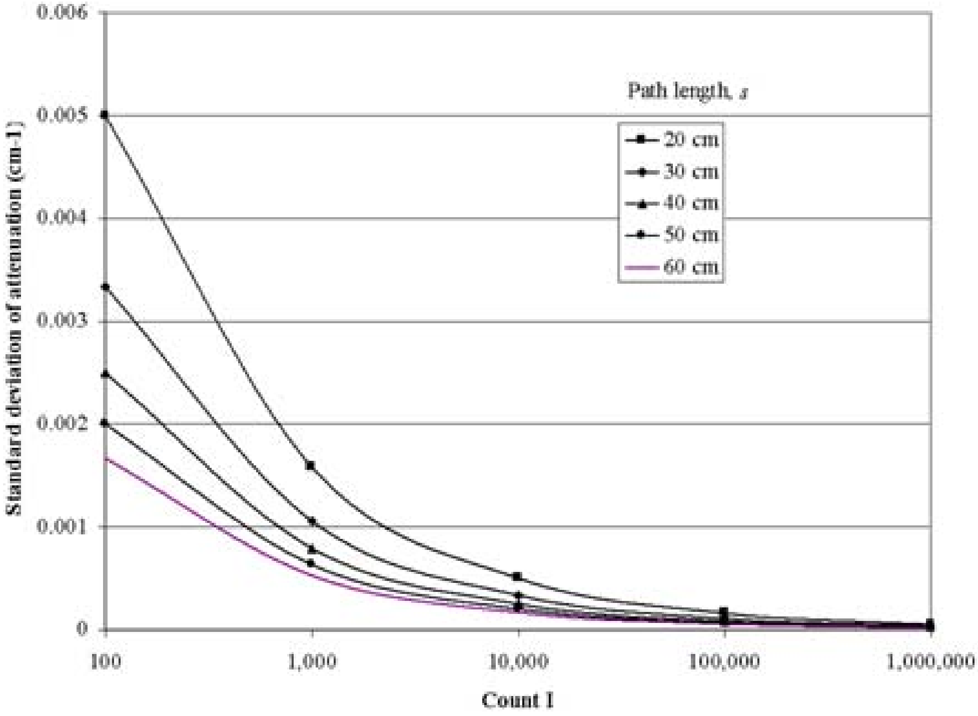
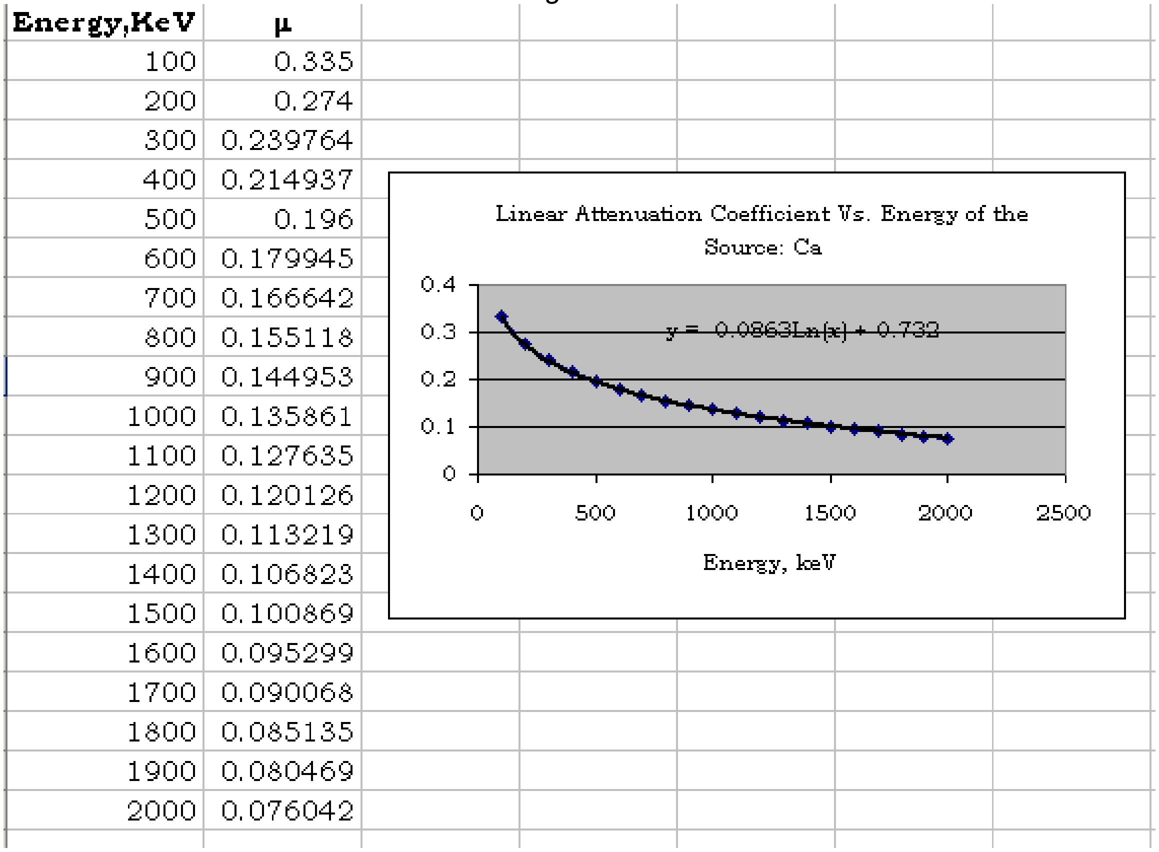


Figure 4.



Energy,KeV	µ
100	0.335
200	0.274
300	0.239764
400	0.214937
500	0.196
600	0.179945
700	0.166642
800	0.155118
900	0.144953
1000	0.135861
1100	0.127635
1200	0.120126
1300	0.113219
1400	0.106823
1500	0.100869
1600	0.095299
1700	0.090068
1800	0.085135
1900	0.080469
2000	0.076042

Figure 5.

$$\text{Precision} \equiv \frac{\sigma(\mu)}{\mu} \cdot 100\% = \frac{[0.0001, 0.0003]}{0.076} \cdot 100\% = [0.13, 0.4]\% \qquad (0.3)$$

In the formula (0.3), in order to be able to use the formula (0.2), the standard deviation of the accumulated number of counts $\sigma(N)$ was substituted with the standard deviation of the attenuation $\sigma(\mu)$.

Therefore, the more detailed **gamma ray densitometry precision** analysis places the precision of light powder gamma ray density measuring devices into an interval of **0.13 – 0.4 %**, corresponding with the precision interval presented in the Schlieper's article [2]. It is important to note that the further increase in the path length will substantially reduce the counts rate value (*n*) due to a limitation on the amount of energy developed at the radioactive source that increases the value of $\sigma(\mu)$, and finally - the precision value. For example, the increase of the path length from 50 cm to 60 cm at 2 MeV energy level of the previous example reduces n from 1000 counts/s to approximately 115 counts/s, bringing $\sigma(\mu)$ to [0.0003, 0.0006] and the precision to
[0.4, 0.8]%.

2. Repeatability of the MPD method

To determine the precision of a particular laboratory sample-MPD device, a representative set of readings of the device's output has to be collected when the device is attached to an empty vessel. In this case, evaluating the precision is equivalent to evaluating the repeatability [1]. We do not have such dataset. The Evonik-conducted experiments resulted in collecting measurement data obtained on the vessel filled to the fullest with Aerosil particles. Therefore, the repeatability value that could be derived based on the Evonik test data will be on the safe lower end (worse) of its possible range because of variations in the powder bulk density. Furthermore, the Evonik-performed test data analysis produced an average percentage of change per g/L of the tested MPD prototype output signal that points at the need of using the "significant figures approach" in evaluating the experimental repeatability of the MPD method: "A common convention in science and engineering is to express accuracy and/or precision implicitly by means of significant figures. Here, when not explicitly stated, the margin of error is understood to be one-half the value of the last significant place" [1]. A more conservative full value of the last significant place is used in the following evaluation. The evaluation will be performed for measurement bulk density of light powder materials lying in the Evonik-defined range of 20 – 150 g/L.

Method 1

1. Low accuracy mechanical and electronic hardware capable of registering up to 1% of the signal.

$$\text{Repeatability} = \frac{\frac{1}{6} g/L}{150 g/L} \cdot 100\% = 0.111\%$$

2. High accuracy electronic hardware capable of registering up to 0.1% of the signal.

   Repeatability $= \dfrac{\frac{1}{60} g/L}{150 g/L} \cdot 100\% = 0.0111\%$

The estimated repeatability lies in the interval between 0.0111% and 0.111% with its average value equal to be 0.061%

Method 2

Assuming that the experimental resolution of the MPD method is of the same order of magnitude as the precision obtained from the case 2 of the Method 1, one will receive:

Repeatability ≈ 5 x Resolution $= 5 \cdot 0.0111\% = 0.0555\%$

3. Conclusion

The repeatability of the MPD method is estimated to be equal to **0.06%**

4. Bibliography

1. Accuracy and Precision/Wikipedia Article
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3. Principles of Gamma Ray Densitometry: Dr. Georg Schlieper/ Metal Powder Report, Volume 55, Issue 12, December 2000, Pages 20-23
4. Basic Physics of Nuclear Medicine/Attenuation of Gamma-Rays/ Wikibooks online article; http://en.wikibooks.org